

# Math 120

## Basic finance percent problems from prior courses (amount = % X base)

- 1) Given a sales tax rate of 8%, a) find the tax on an item priced at \$250, b) find the total amount due (which includes both the price and the tax).
- 2) Given a sales tax rate of 7 %, a) find the price of an item if the total, including tax, equals \$355.24, b) what is the amount of the tax?
- 3) If 6% sales tax on an item is \$13.68, find the price of the item.
- 4) If the total including sales tax is \$439.93, but you know the tax is \$29.93, find the sales tax rate.
- 5) Find the commission on a \$4,500 sale, if the commission rate is 2.2% .
- 6) Find the sale, given the 8% commission is \$3,240.
- 7) Find the percent commission on an item if the sales price is \$4,300 and the commission is \$275.20.

- 8) Find the amount of a discount if the normal price of an item is \$600 and there is a 30% discount.
- 9) Find the original price of an item if the final cost after a 70% discount is \$2,460.
- 10) What is the original price of an item if a 20% discount gives you a \$560 savings?
- 11) Find the percent discount if the original price is \$820 and the price after the discount is \$598.60.
- 12) Find the new salary after a 10% pay increase if the salary before the increase was \$2,430 per month.
- 13) Find the old salary if the pay after an 8% increase is \$783.54.
- 14) If there is an item whose original price is \$540, but which is on sale for a 25% discount, what is the total paid if there is a 7% sales tax on the amount actually paid for the item?

# Answers to Basic finance percent problems

- 1) a) Tax = % \* price  
Tax = 8% \* \$250  
Tax = \$20.00
- b) Total = price + tax  
Total = \$250 + \$20  
Total = \$270.00  
or Total = (100% + tax %) \* price  
Total = 108% \* \$250  
Total = \$270.00
- 2) a) Total = (100% + tax %) \* price  
\$355.24 = 107% \* price  
Price = \$332.00
- b) Total = price + tax  
\$355.24 = \$332 + tax  
Tax = \$23.24
- 3) Tax = % \* price  
\$13.68 = 6% \* price  
Price = \$ 228.00
- 4) Price = total – tax  
Price = \$439.93 - \$29.93  
Price = \$410.00
- Tax = % \* price  
\$29.93 = % \* \$410  
% = 7.3%
- 5) Commission = % \* sale  
Commission = 2.2% \* \$4,500  
Commission = \$99.00
- 6) Commission = % \* sale  
\$3,240 = 8% \* sale  
Sale = \$40,500.00
- 7) Commission = % \* sale  
\$275.20 = % \* \$4,300  
% = 6.4%

- 8) Discount = % \* price  
Discount = 30% \* \$600  
Discount = \$180.00
- 9) Cost = (100% - discount %) \* price  
\$2,460 = 30% \* price  
Price = \$8,200.00
- 10) Discount = % \* price  
\$560 = 20% \* price  
Price = \$2,800.00
- 11) Cost = price - discount  
Cost = \$820 - \$598.60  
Cost = \$221.40  
Discount = % \* price  
\$221.40 = % \* \$820  
% = 27%
- 12) New = (100% + increase %) \* old  
New = 110% \* \$2,430  
New = \$2,673.00
- 13) New = (100% + increase %) \* old  
\$783.54 = 1.08 \* old  
Old = \$725.50
- 14) Cost = (100% - Discount %) \* price  
Cost = 75% \* \$540  
Cost = \$405.00  
Total = (100 % + tax %) \* cost  
Total = 107 % \* \$405  
Total = \$ 433.35

# COMPOUND INTEREST

## VARIABLES

$P$  = Present Value (or Principal) – value before compounding

$F$  = Future Value – value after compounding

$A$  = Annuity Value – value added or subtracted at END of each time period

$t$  = Number of Time Periods (most often number of years)

$r$  = Interest Rate per time period (most often one year)

$r_{\text{nom}}$  = Stated (Nominal) Interest Rate

$r_{\text{eff}}$  = Effective Interest Rate

Note: \* will be used for multiplication

# COMPOUND INTEREST

## Comparison of Simple and Compound interest

Simple Interest:  $F = P * (1 + rt)$ , i.e. the same interest is added each year

Compound Interest:  $F = P * (1 + r)^t$ , i.e. each year the former year's amount is multiplied by {one plus the interest rate}

Example: Invest \$100 for 4 years at 10% annual interest

<u>Years Loaned (time)</u>	<u>Value @ Simple Interest</u>	<u>Value @ Compound Interest</u>
0	\$ 100.00	\$ 100.00
1	\$ 110.00	\$ 110.00
2	\$ 120.00	\$ 121.00
3	\$ 130.00	\$ 133.10
4	\$ 140.00	\$ 146.41

Note that with simple interest, 10% of the original principal (\$100) was added on each year. With compound interest, the value at the end of the year was 110% of the amount the year before (100% for what was already owed plus 10% for interest.)

# COMPOUND INTEREST

INTEREST: EFFECTIVELY HIGH

## EFFECTIVE INTEREST

Change annual interest rate ( $r_{\text{nom}}$ ) to a decimal

Divide annual interest rate ( $r_{\text{nom}}$ ) by number of time periods ( $n$ )  
(12 for monthly, 365 for daily, etc.)

Add one

Take to power of number of times periods (i.e. 12 if monthly)

Subtract one and change to back to percent

Formula is then:  $r_{\text{eff}} = \{1 + (r_{\text{nom}} / n)\}^n - 1$

Example: 12% nominal interest rate compounded monthly

$$r_{\text{nom}} / n = 12\% / 12 = .12 / 12 = .01 \text{ per month}$$

$$1 + r_{\text{nom}} / n = 1 + .01 = 1.01$$

$$\{1 + r_{\text{nom}} / n\}^n = 1.01^{12} = 1.1268$$

$$r_{\text{eff}} = \{1 + r_{\text{nom}} / n\}^n - 1 = 1.1268 - 1 = .1268 = 12.68\%$$

On the calculator, do the following:

$$.12 / 12 = + 1 = y^x 12 = - 1 =$$

A single formula for finding the future value using effective interest over a number of time periods is:

$$F = P * (1 + r_{\text{nom}} / n)^{nt}$$

# COMPOUND INTEREST

## BASIC FORMULAS: PRESENT/FUTURE VALUES AND ANNUITIES

$$F = P * (1 + r)^t$$

$$P = F / (1 + r)^t$$

$$F = A * \{(1 + r)^t - 1\} / r$$

$$A = F * r / \{(1 + r)^t - 1\}$$

$$P = A * \{(1 + r)^t - 1\} / \{r*(1 + r)^t\} \quad A = P * \{r*(1 + r)^t\} / \{(1 + r)^t - 1\}$$

Example: Using 10% annual interest, with \$ 1,000 invested at the END of each of 7 years, find the future value (A = \$ 1,000, r = 10%, t = 7 years)

$$\begin{aligned} F &= A * \{(1 + r)^t - 1\} / r \\ &= 1000 \{(1.10)^7 - 1\} / .10 \\ &= 1000 \{1.9487171 - 1\} / .1 \\ &= 1000 (9.487171) \\ &= \underline{\underline{\$ 9,487.17}} \end{aligned}$$

Checking this manually: at the end of each time point (year):

Year	Value @ Beginning	With 10% Interest	Add \$1,000	Final Value
1	\$ 0	\$ 0	\$ 1,000	\$ 1,000
2	\$ 1,000	\$ 1,100	\$ 1,000	\$ 2,100
3	\$ 2,100	\$ 2,310	\$ 1,000	\$ 3,310
4	\$ 3,310	\$ 3,641	\$ 1,000	\$ 4,641
5	\$ 4,641	\$ 5,105.10	\$ 1,000	\$ 6,105.10
6	\$ 6,105.10	\$ 6,715.61	\$ 1,000	\$ 7,715.61
7	\$ 7,715.61	\$ 8,487.17	\$ 1,000	\$ 9,487.17

Example: For a present value (P) of \$20,000, r = 8%, and t = 20 years, find the annuity amount (A).

$$\begin{aligned} A &= P * \{r*(1 + r)^t\} / \{(1 + r)^t - 1\} \\ &= (20,000) * \{.08 * (1.08)^{20}\} / \{(1.08)^{20} - 1\} \\ &= (20,000) * \{.08 * (4.660957144)\} / \{4.660957144 - 1\} \\ &= (20,000) * \{.10204045\} \\ &= \$ 2,037.04 \end{aligned}$$

# COMPOUND INTEREST

## Finding Future/Present Values, Time, Interest Rate

$$F = P (1 + r)^t$$

$$P = F / (1 + r)^t$$

$$t = \log_{1+r} (F/P)$$

$$r = (F/P)^{1/t} - 1$$

As an example, let's assume we are given any 3 of the 4 variables in the following scenario:

\$100,000 is placed into an investment at an interest rate of 9.23 %, compounded annually. The value at the end of 7.8 years is \$199,098.08

So, the values are:

$P = \$100,000$
$F = \$199,098.08$
$r = .0923$
$t = 7.8$

To find each value (rounded), given the other 3:

### **To find Future Value (F)**

$$\begin{aligned} F &= P (1 + r)^t \\ &= 100,000(1.0923)^{7.8} \\ &= \$199,098.08 \end{aligned}$$

### **To Find Present Value (P)**

$$\begin{aligned}P &= F / (1 + r)^t \\ &= \$199,098.08 / (1.0923)^{7.8} \\ &= \$100,000\end{aligned}$$

### **To find time**

$$\begin{aligned}t &= \log_{1+r} (F/P) \\ &= \log_{1.0923} (199,098.08 / 100,000) \\ &= \log_{10} (1.9909808) / \log_{10} (1.0923) \\ &\quad \text{("log" on calculator)} \\ &\text{or } \log_e (1.9909808) / \log_e (1.0923) \\ &\quad \text{("ln" on calculator)} \\ &= 7.8 \text{ years}\end{aligned}$$

### **To find Interest rate**

$$\begin{aligned}r &= (F/P)^{1/t} - 1 \\ &= (199,098.08 / 100,000)^{1/7.8} - 1 \\ &= 9.23 \%\end{aligned}$$

# COMPOUND INTEREST

## MORTGAGE ASSIGNMENT

This is a contrived assignment (to make it easier) for a mortgage-type loan. Interest rate will be 8%, compounded and paid quarterly, on a one-year, \$10,000 loan. You are to fill in the table completely. Assume you borrowed \$10,000 to buy a motorcycle, and want it totally paid off after one year.

Qtr	Beginning Balance	Total Payment	Interest Paid	Principal Paid	New Balance
1	\$10,000				
2					
3					
4					0 (+- pennies)

To start this assignment, figure out the quarterly payment using the formula

$$A = P \left\{ r (1+r)^n / [ (1+r)^n - 1 ] \right\}, \text{ using } P = \$10,000, r = 2\%, n = 4$$

That amount becomes the total payment each quarter, and does not change. The interest equals 2% of the quarter's beginning balance. Subtracting the interest from the payment gives the principal paid. The Beginning balance minus the principal paid equals the new balance, and that becomes the next quarter's beginning balance.

The final balance should equal zero, except for rounding. Please round all entries to the nearest cent.

# COMPOUND INTEREST

## RULE OF 72 (compound interest)

For an approximation of the number of years required to double an investment, divide 72 by the annual interest rate in percent. To find the interest rate required to double in a specific number of years, divide 72 by the number of years.

Example: at 9 % annual interest, it should take about  $72 / 9$  or eight years to double (the actual amount after 8 years would be 1.99256 times the original amount).

Example: to double an investment in 12 years, the interest rate should be about  $72 / 12$  or 6 % (after 12 years at 6 % interest compounded annually, the actual amount would be 2.0122 times the original amount.)

Example: at 12 % annual interest, it should take about  $72 / 12$  or six years to double (the actual amount after 6 years would be 1.9738 times the original amount).

This approximation works very well up to approximately 24 %. At 24 %, the actual investment would be 1.9066 times the original investment after three years.

Exercises (Round all answers to the nearest cent, or to the nearest hundredth of a percent, where applicable.)

- 1) To help pay her tuition bill, Sally borrowed \$1,200 at  $9\frac{1}{2}\%$  simple interest for 4 years.
  - a) How much interest will she owe after 4 years?
  - b) What is the total amount she will owe?
- 2) Using 5% annual simple interest, how many years will it be before \$2,000 is worth \$3,500?
- 3) Using 8 % annual compound interest, what will an investment of \$2,000 be worth at the end of 7 years?
- 4) A \$10,000 loan is made at 11 % annual interest to be paid in one lump sum after 20 years. Find the total owed:
  - a) Using simple interest
  - b) Using compound interest
- 5) Having borrowed \$10,000 at 9 % annual interest, compounded monthly, how much will need to be paid back in one lump sum after 8 years?
- 6) If a credit card charges a nominal rate of 18%, what is the effective rate: assume interest is compounded monthly?
- 7) If the nominal annual rate of interest is 6%, find the effective rate if compounded weekly (use 52 weeks / year).
- 8) Calculate the total amount of interest on a \$2,000 loan at 4% annual interest for 3 years:
  - a) using simple interest
  - b) if the interest is compounded annually
  - c) if the interest is compounded quarterly

- 9) If you have \$5,000 to invest for 2 years, which investment plan is best (Justify your answer with proper calculations and explanations)?
- a) 8% simple annual rate
  - b) 7.8% rate with interest compounded annually
  - c) 7.6% annual rate with interest compounded quarterly
- 10) If you borrow \$20,000 at 8% annual interest, compounded monthly, to be paid back in equal payments over ten years, what is your monthly payment?
- 11) How much would a monthly investment of \$200 (at the end of each month) be worth after 20 years at:
- a) 3 % nominal annual interest?
  - b) 6 % nominal annual interest?
  - c) 9 % nominal annual interest?
  - d) 12 % nominal annual interest?
  - e) Subtract the total principal invested (\$200 per month times 12 months per year times 20 years for a total of \$48,000) and compare the total interest in each part above.
- 12) How much would a monthly investment of \$500 (at the end of each month) be worth assuming 6 % compound annual nominal interest?
- a) after 10 years
  - b) after 20 years
  - c) after 30 years
  - d) after 40 years
  - e) Why is the difference between answers so large?

- 13) Assume that \$100,000 is needed ten years from today, and that you can get an interest rate of 8 %, compounded annually.
- a) How much would you have to invest at the end of each year?
  - b) How much would you have to put aside one year from today to have that amount? (Note that you would only have nine years of growth.)
- 14) Assuming that \$200,000 is needed ten years from today, and that you can get an annual interest rate of 12 %, compounded monthly.
- a) How much would you have to invest at the end of each month to have that amount?
  - b) How much would you have to put aside today to have that amount ?
- 15) Assume a 6 % annual interest rate, compounded monthly: if \$500 is placed in an account at the end of each month for  $2\frac{1}{2}$  years.
- a) What is the present worth?
  - b) Give an example as to why you might be interested in the present worth.
  - c) What is the future worth?
  - d) Give an example as to why you might be interested in the future worth.

- 16) In February of 2010, a credit card for bad credit was introduced with a nominal interest of 79.9% (that is NOT a typo: 79.9 %). Assume that interest on a credit card is compounded monthly, and that a \$1,000 purchase is made:
- What is the effective interest rate?
  - What monthly amount would be required for the bill to be paid off in 10 years?
  - Find the total of all payments in part b)?
  - If for some reason all payments were deferred (no penalties, etc) for 10 years, what would be the amount owed?
- 17) For some reason an investment is made using annual compound interest which turns \$10,000 invested into \$25,000.
- How many years (to the nearest tenth) will it take if the interest rate is 8%?
  - What interest rate is required to finish the plan in 10 years?
- 18) What annual compound interest rate is required for a \$40,000 investment to be worth \$50,000 in 3 years?
- 19) At 5% annual compound interest, how long would it take, to the nearest tenth of a year, for a one-time investment of \$5,000 to become worth \$14,200?
- 20) For \$10,000 to become \$15,000 after 5 years:
- Before doing the calculations, would you expect a lower simple or compound interest rate would be required to make it happen?
  - What simple interest rate will it take?
  - What compound interest rate will it take?

21) The “Rule of 72” states: “ In compound interest, the nominal \_\_\_\_\_ rate times the number of \_\_\_\_\_ to \_\_\_\_\_ approximates 72.”

22) Using the “Rule of 72”:

- a) About how many years will it take an amount to double with an interest rate of 7.2 % ?
- b) What annual compound interest is required for an amount to double in about 18 years?

23) How many years are needed for \$1,000 to become \$8,000 at 9% interest, compounded annually:

- a) Using the “Rule of 72” ?
- b) Exactly ?
- c) How close are the answers above?

24) If you take out a standard mortgage of \$ 300,000 for 15 years with an interest rate of 9%:

- a) Determine your monthly principal and interest.
- b) Then fill-in the first 2 months of the Amortization schedule:

<u>Time</u>	<u>Old Balance</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal</u>	<u>New Balance</u>
0	-	-	-	-	\$300,000.00
1	\$300,000.00				
2					

25) If you take out a standard mortgage of \$200,000 for 30 years at an annual rate of 6%:

- a) Determine your monthly principal and interest.
- b) Write the amortization statement for the first four months of the loan.
- c) Write the amortization statement for the 99<sup>th</sup> and 100<sup>th</sup> months of the loan (the balance at the end of the 98<sup>th</sup> month is \$174,900.35)

## Answers

- 1) a) \$456  
b) \$1,656
- 2) 15 years
- 3) \$3,427.65
- 4) a) \$ 32,000.00  
b) \$ 80,623.12
- 5) \$20,489.21
- 6) 19.56 %
- 7) 6.18 %
- 8) a) \$240  
b) \$249.73  
c) \$253.65
- 9) 7.6 % compounded quarterly is the best. Total value:  
a) \$5,800.00  
b) \$5,810.42  
c) \$5,812.51
- 10) \$242.66
- 11) a) \$ 65,660.40  
b) \$ 92,408.18  
c) \$ 133,577.37  
d) \$ 197,851.07  
e) Interest:     3%   \$ 17,660.40  
                  6%   \$ 44,408.18  
                  9%   \$ 85,577.37  
                  12%  \$ 149,851.07

The interest at 12% is about 8 ½ times 3 %:  
compound interest at work. Total  
interest rises significantly in all cases.

- 12) a) \$ 81,939.67  
b) \$ 231,020.45  
c) \$ 502,257.52  
d) \$ 995,745.37  
e) Magic of compound interest: time value of money

- 13) a) \$ 6,902.95  
b) \$ 50,024.90 (note similarity to “Rule of 72”)
- 14) a) \$ 869.42  
b) \$ 60,598.96
- 15) a) \$ 13,897.03  
b) To find how much could you borrow to be paid back at \$500 per month (example: mortgage)  
c) \$16,140.01  
d) Find out how much you have to spend in 10 years
- 16) a) 116.74%  
b) \$66.61  
c) \$7,993.20  
d) \$2,287,574.05
- 17) a) 11.9 years  
b) 9.60%
- 18) 7.72 %
- 19) 21.4 years
- 20) a) Compound, since the loan is for more than 1 year  
b) 10 %  
c) 8.45 %
- 21) The “Rule of 72” states: “In compound interest, the nominal annual interest rate times the number of years to double approximates 72.”
- 22) a) 10 years  
b) 4 %
- 23) a) 24 years  
b) 24.1 years  
c) one-tenth of a year (actually closer to .13 year)

24) Monthly payment: \$ 3,042.80

<u>Time</u>	<u>Old Balance</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal</u>	<u>New Balance</u>
0	-	-	-	-	\$300,000.00
1	\$300,000.00	\$3,042.80	\$2,250.00	\$792.80	\$299,207.20
2	\$299,207.20	\$3,042.80	\$2,244.05	\$798.75	\$298,408.45

25) Monthly payment: \$1,199.10

<u>Time</u>	<u>Old Balance</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal</u>	<u>New Balance</u>
0	-	-	-	-	\$200,000.00
1	\$200,000.00	\$1,199.10	\$1,000.00	\$199.10	\$199,800.90
2	\$199,800.90	\$1,199.10	\$ 999.00	\$200.10	\$199,600.80
3	\$199,600.80	\$1,199.10	\$ 998.00	\$201.10	\$199,399.70
4	\$199,399.70	\$1,199.10	\$ 997.00	\$202.10	\$199,197.60
99	\$174,900.35	\$1,199.10	\$ 874.50	\$324.60	\$174,575.75
100	\$174,575.75	\$1,199.10	\$ 872.88	\$326.22	\$174,249.53