

# LOGARITHMS

1) Functions:  $y = f(x)$

Each “x” goes to exactly one “y” (vertical line test)

One-to-one function:  $y = fcn(x)$  and  $x = fcn(y)$

Vertical and horizontal line tests

Inverse of function:  $y = f(x)$ ,  $x = f^{-1}(y)$

Must be one-to-one

2) Exponential Function:  $y = b^x$  (where  $b$  is a constant)

One-to-one

Domain:

Range:

3) Definition of Logarithm: (inverse of exponential function)

“ For  $b > 0$  and  $b \neq 1$ , if  $x = b^y$  then  $y = \log_b x$  “

4) Example: (with base = 2)

$$8 = 2^3 \quad \text{so } \log_2 8 = 3$$

$$1024 = 2^{10} \quad \text{so } \log_2 1024 = 10$$

$$2 = 2^1 \quad \text{so } \log_2 2 = 1$$

$$1 = 2^0 \quad \text{so } \log_2 1 = 0$$

$$1/16 = 2^{-4} \quad \text{so } \log_2 (1/16) = -4$$

5) Domain of Logarithm:  $(0 < x < \infty)$

$$y = \log_b x \text{ means } x = b^y \text{ (b positive),}$$
$$\text{so } x = \text{any real number } > 0$$

6) Range of Logarithm:  $(-\infty < y < \infty)$

$$y = \log_b x \text{ means } x = b^y \text{ (b positive),}$$
$$\text{so } y = \text{any real number}$$

7) Special Notes:

$$\log_b 1 = 0 \quad (\text{since } b^0 = 1)$$
$$\log_b b = 1 \quad (\text{since } b^1 = b)$$

8) Natural Base “e”,  $e \approx 2.71828$

9)  $\text{Log}_b b^x = x$

$$b^{\log_b x} = x$$

10) Change of Base

$$\text{Log}_a (N) = \log_b (N) / \log_b (a)$$

$$\text{Example: } \text{Log}_8 64 = \log_2 (64) / \log_2 (8) = 6/3 = 2$$

$$\text{Note: Also } \text{Log}_8 64 = 2 \quad \text{because } 8^2 = 64$$

11) Basic Scientific Calculator

LOG is base "10" ( $\text{Log}_{10}$ )

LN is base "e" ( $\text{Log}_e$ )

12) Example: Richter Scale (Earthquakes)

Magnitude of the Quake (M) =  $\log_{10}(T)$

Where T = amplitude of the shock wave

So: A magnitude 3 earthquake has 10 times  
the amplitude of a magnitude 2 quake

## Exercises: (remember definitions)

a) Graph  $y = 2^x$  and  $y = \log_2 x$

(suggest making tables for  $x$  versus  $2^x$  and  $x$  versus  $\log_2 x$ )

b) Put  $64 = 4^3$  in log form      Put  $128 = 2^7$  in log form

c) Find  $\log_2 (1/8)$       Find  $\log_{10} (100,000)$

d) If  $\log_2 (x) = 5$ , find  $x$

e) Write in Log form:       $4^{-2} = 1/16$        $B^X = T$

f) Write in Exponential form:  $\log_{10} (100) = 2$

$\log_2 (1/32) = -5$        $\log_{.5} (1/32) = 5$

g) Evaluate:  $\text{Log}_5 125$        $\text{Log}_{10} 100,000,000$

h) Solve for X:  $\text{Log}_3 X = 1$        $\text{Log}_2 X = 4$

$\text{Log}_3 X = 0$        $\text{Log}_7 X = -2$

i) Find  $\log_{18.3} 18.3^{19}$       Find  $7^{\log_7 8}$

j) Use a scientific calculator to determine (round):

$\text{Log}_{10} (10)$        $\text{Log}_e (5)$

$\text{Log}_5 (20)$

Hint: use  $\text{Log} (20) / \text{Log} (5)$  or  $\text{Ln} (20) / \text{Ln} (5)$

$\text{Log}_8 (1/64)$        $\text{Log}_{3.5} (.5)$

$\text{Log}_7 (700)$        $\text{Log}_7 (343)$

$\text{Log}_2 (8)$        $\text{Log}_2 (1/8)$

$\text{Log}_{.5} (1/8)$        $\text{Log}_{.5} (8)$