Logic

Two basic types of logic:

Inductive

Specific to General: example ____

Based on Probability

Deductive

General to Specific: example ____

Always True

Statements (Sentences/Propositions), generally represented by P, Q, R, etc

TRUE or FALSE (Not subjective)

A Question is not a statement

A Paradox is not a statement ____

Simple

Compound (Complex)

Basic Symbols – note similarity to Set Theory

Not	~	Negation
And	Λ	Conjunction
Or (default)	V	Disjunction
Exclusive OR	$\underline{V}$	
If…Then (Implies)	→	Conditional
If and Only If (iff)	↔	Biconditional
Equivalent to	Ξ	(Note similarity to equals)

Dominance of Connectives (Parentheses sometimes make it clear)

↔ → Λ V $\underline{V}$ ~

Truth Tables

P	Q	~P	~Q	PΛQ	PVQ	P$\underline{V}$Q	~(PΛQ)	~(PVQ)	~(P$\underline{V}$Q)	PΛ~Q
T	T	F		T	T		F			
T	F	F		F	T		T			
F	T	T		F	T		T			
F	F	T		F	F		T			

P	Q	R	~P	~Q	~R	PΛQΛR	PVQVR	~PΛ~QΛ~R	~PV~QV~R
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

P	Q	R	~Q	~R	PΛ~QΛR	PΛQ	~(PΛQ)	~(PΛQ)V~R
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

P	Q	$P \to Q$	$P \leftrightarrow Q$	$\sim P$	$\sim P \vee Q$	$(P \to Q) \leftrightarrow (\sim P \vee Q)$
T	T	T	T			
T	F	F	F			
F	T	T	F			
F	F	T	T			

Note: does this mean $(P \to Q) \equiv (\sim P \vee Q)$?

DeMorgan' Laws

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$P \vee Q$	$\sim(P \wedge Q)$	$\sim P \vee \sim Q$	$\sim(P \vee Q)$	$\sim P \wedge \sim Q$
T	T								
T	F								
F	T								
F	F								

$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$ $\qquad$ $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Note similarity to Venn Diagrams (DeMorgan's Laws)

Equivalent Statements: (Reference Section 1.4)

P	Q	$\sim P$	$\sim Q$	Statement $P \to Q$	Converse $Q \to P$	Inverse $\sim P \to \sim Q$	Contrapositive $\sim Q \to \sim P$
T	T						
T	F						
F	T						
F	F						

Statement $P \to Q$ equivalent to $\equiv$ Contrapositive $\sim Q \to \sim P$

Converse $Q \to P$ equivalent to $\equiv$ Inverse $\sim P \to \sim Q$

P	Q	P→Q	Q→P	(P→Q)∧(Q→P)	P ↔ Q	(P→Q)∧(Q→P)↔(P ↔ Q)
T	T					
T	F					
F	T					
F	F					

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P	Q	R	QVR	P∧Q	P∧R	P∧(QVR)	(P∧Q)V(P∧R)	P∧(QVR) ↔ (P∧Q)V(P∧R)
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

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P	Q	R	Q∧R	PVQ	PVR	PV(Q∧R)	(PVQ)∧(PVR)	PV(Q∧R) ↔ (PVQ)∧(PVR)
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

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Question: Are the following statements true?

P∧(QVR) Ξ (P∧Q)V(P∧R)

PV(Q∧R) Ξ (PVQ)∧(PVR)

If they are true, do they look somewhat like the Distributive Laws of Multiplication over Addition and Subtraction? If so, what might be a good name for them?

ARGUMENTS – VALID OR INVALID

Arguments are valid if the logic is correct. We start with "premises" (assumptions). A valid argument logically leads from the premises to the conclusion. The conclusion is only necessarily true if both the logic is valid and the assumptions are correct.

An example would be:

Premises:

In a certain course, achieving at least 90% of the total points means a grade of "A".

You received 92% of the points in the course.

Conclusion:

Therefore, you will receive an "A" in the class.

This is a valid conclusion.

An example of a valid argument with a false conclusion might be:

Premises:

The total cost of lunch with tip, tax, etc is $10.92

(This could be restated as: "if you have at least $10.92 in your pocket, then you have enough money for lunch.")

You have 8 million dollars in your pocket

Conclusion:

Therefore, you have money for lunch.

This is also a valid conclusion, but it may not be true, since at least one of the premises is quite possibly wrong (most of us don't carry 8 million dollars in our pockets.)

In both examples, the logic comes directly from the following form:

$(P \rightarrow Q) \wedge P \quad \therefore Q$ The symbol "$\therefore$" is often used for the word "therefore".

Technically, that's $(P \rightarrow Q) \wedge P \rightarrow Q$

----- Premises:

If P, then Q (i.e. P implies Q).

Also, P happens.

Conclusion:

Therefore ($\therefore$), Q must happen.

You should be able to prove this form { $(P \rightarrow Q) \wedge P \therefore Q$ } using both Venn Diagrams and Truth Tables

VALID FORMS OF AN ARGUMENT

The most common valid forms of an argument are:

$(P \rightarrow Q) \wedge P$	$\therefore Q$	This is sometimes called "modus ponens"
$(P \rightarrow Q) \wedge \sim Q$	$\therefore \sim P$	Called "modus tollens", this is the contrapositive
$(P \rightarrow Q) \wedge (Q \rightarrow R)$	$\therefore P \rightarrow R$	"hypothetical syllogism" – not actual syllogism
$(P \vee Q) \wedge \sim P$	$\therefore Q$	"disjunctive syllogism" – not actual syllogism

In addition, true syllogisms are used: (Venn Diagrams are usually easier here)

Universal Affirmative: All P are Q

The classic example of a Universal Affirmative syllogistic argument is given by:
All men are mortal
Socrates is a man
∴Socrates is a mortal

(Isn't this really the same as $(P \rightarrow Q) \wedge P \quad \therefore Q$?)

Universal Negative: No P are Q i.e. $P \rightarrow \sim Q$

Particular Affirmative: Some P are Q i.e. "there exists at least one"

Particular Negative: Some P are not Q

The different forms of an argument may be used together to form a "thesis". For this course, it is not important to know the names of the different forms of a valid argument: they are presented here for those who may need them outside this class.

FALLACIES

Fallacies are basically invalid arguments. The most common are very easy to mistake (hence the most common):

a) $(P \rightarrow Q) \wedge Q \quad \therefore P$ Affirming the consequent or fallacy of the converse

EXAMPLE If one is a dog, then he is a mammal
You are a mammal
Therefore, you are a dog

b) $(P \rightarrow Q) \wedge \sim P \quad \therefore \sim Q$ Fallacy of the inverse

EXAMPLE If one is a dog, then he is a mammal
You are not a dog
Therefore, you are not a mammal

EXERCISES:

1) Prove: $(P \rightarrow Q) \wedge (Q \rightarrow R)$ $\therefore$ $P \rightarrow R$ i.e. $\{(P \rightarrow Q) \wedge (Q \rightarrow R)\} \rightarrow (P \rightarrow R)$

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	$P \rightarrow R$	$\{(P \rightarrow Q) \wedge (Q \rightarrow R)\} \rightarrow (P \rightarrow R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

2) Prove: $\sim(P \underline{\vee} Q) \equiv P \leftrightarrow Q$

P	Q	$P \underline{\vee} Q$	$\sim(P \underline{\vee} Q)$	$P \leftrightarrow Q$	$\sim(P \underline{\vee} Q) \equiv P \leftrightarrow Q$

3) Give an example of a simple statement.

4) Give 3 examples of compound statements.

5) State a paradox.

6) Describe how set theory (Venn Diagrams) and truth tables are related.

7) Give an example using the “exclusive or”.

8) Give an example using the “inclusive or”.

9) Write the following statements in proper symbolic form (label what P , Q mean)

   a) I’ll go to lunch only if I have the cash

   b) I’ll go to breakfast if I have the cash

   c) I’ll go to dinner iff I have the cash

10) Assume the statement: “green vittles are delicious” is true.
   a) Decide which of the following statements are necessarily true
   b) Label each statement as converse, inverse, contrapositive

Delicious vittles are green ____

Non-delicious vittles are not green ____

Non-green vittles are not delicious ____

11) State one of DeMorgan’s laws and prove it by using Venn diagrams.

12) State the other one of DeMorgan’s laws and prove it by using truth tables.

13) Are the following arguments valid? If so, give the logic form (do not state the name of the form) or show via Venn diagrams or truth tables. If not, describe why.

a) All colleges are expensive
CSN is expensive
Therefore, CSN is a college ____

b) Some rectangles are squares
Some squares are big
Therefore some rectangles are big __________________________________

c) When I meet with Joe, he is always late
I'm tired of waiting for the meeting – he is late again
Therefore the meeting is with Joe __________________________________

d) All Philosophers are smart
Smart people are quirky
Fred is not quirky
Therefore, Fred is not a philosopher ________________________________

e) DeMorgan's laws are easy to use
These laws are not DeMorgan's
Therefore, they are not easy to use _________________________________

f) If the letter T is more sublime than the letter Q, and the letter R is more sublime than the letter Q, then the letter R is more sublime than the letter T

____

g) You cannot have super-strength without Power Powder. Therefore you will have super-strength with Power Powder

____

h) Bill grows asparagus and broccoli
Bill grows broccoli
Therefore he does not grow asparagus _____________________________

i) Joan grows apples or peaches
Joan does not grow peaches
Therefore Joan grows apples ______________________________________

j) Linda grows mushrooms or flowers
Linda grows mushrooms
Therefore, Linda does not grow flowers _____________________________

k) In the horsey system of Arithmetic, addition is the same as we use in the real number system
We are not using the horsey system of Arithmetic
Therefore, in the system we are using, $4 + 4 \neq 8$ _______________________

l) Not all students in a Chemistry class will receive an A
No student in the class received an A, except for maybe Jill
Therefore, Jill received an A ____

m) All flowers are aromatic
No sunflowers are aromatic
Therefore, No sunflowers are flowers _______________________________

n) You are crazy and wild
You are crazy
Therefore you are wild ____

o) Mathematics is fun or difficult
Mathematics is fun
Therefore it is difficult ____

p) Mathematics is fun or difficult
Mathematics is fun
Therefore it is not difficult ____

q) If it is raining outside, Joan will take her umbrella
Joan takes her umbrella
Therefore, it is raining outside _______________________________________

r) If Tom is upset, then Julie is eating
Julie is not eating
Therefore, Tom is not upset ____

s) If Quartzes are Diamonds, then goods are extraneous
If goods are extraneous, then gallops are geographic
Gallops are not geographic
Therefore Quartzes are not Diamonds _______________________________

t) Some horses are champions
Buster is not a horse
Therefore, Buster is not a champion _________________________________

u) No teddies are bulls
Joe is a bull
Therefore, Joe is not a teddie ______________________________________

v) Hooch is not a happy hold or gooch is not a gappy gold
Therefore hooch is not a happy hold and gooch is not a gappy gold ______

w) A→B, B→C, C→D
Therefore A→D ____

14) State the four most common valid forms of an argument (4)

15) State the four true syllogisms (4)

16) State and name the two most common fallacies

ANSWERS:

1) Prove: $(P \to Q) \land (Q \to R)$ $\therefore P \to R$ i.e. $\{(P \to Q) \land (Q \to R)\} \to (P \to R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \land (Q \to R)$	$P \to R$	$\{(P \to Q) \land (Q \to R)\} \to (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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2) Prove: $\sim(P \underline{\vee} Q) \equiv P \leftrightarrow Q$

P	Q	$P \underline{\vee} Q$	$\sim(P \underline{\vee} Q)$	$P \leftrightarrow Q$	$\sim(P \underline{\vee} Q) \equiv P \leftrightarrow Q$
T	T	F	T	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

3) Blue is a color.
4) Blue is not a color.
   Greed is good and fortunes are fabulous.
   If green is red and good is bad, then 1 + 1 = 2.
5) This statement is a lie.
6) Your ideas.
7) I am tired or hungry, but not both.
8) I have money for lunch, or you have money for lunch.
9)
   a) P = lunch, Q = cash; $P \to Q$
   b) P = breakfast, Q = cash; $Q \to P$
   c) P = dinner, Q = cash; $P \leftrightarrow Q$
10)
   Delicious vittles are green ________False Converse________
   Non-delicious vittles are not green ________True Contrapositive________
   Non-green vittles are not delicious ________False Inverse________

11) $\sim(P \lor Q) \equiv \sim P \land \sim Q$ Your proof

12) $\sim(P \land Q) \equiv \sim P \lor \sim Q$ Your proof

13) You must describe why yourself

a) Not valid
b) Not valid
c) Not valid
d) Valid
e) Not valid
f) Not valid
g) Not valid
h) Not valid
i) Valid
j) Not valid
k) Not valid
l) Not valid
m) Valid
n) Valid
o) Not Valid
p) Not Valid
q) Not Valid
r) Valid
s) Valid
t) Not Valid
u) Valid
v) Not Valid
w) Valid

14) Valid forms of an argument

$(P \rightarrow Q) \land P$	$\therefore Q$
$(P \rightarrow Q) \land \sim Q$	$\therefore \sim P$
$(P \rightarrow Q) \land (Q \rightarrow R)$	$\therefore P \rightarrow R$
$(P \lor Q) \land \sim P$	$\therefore Q$

15) Syllogisms

Universal Affirmative:	All P are Q
Universal Negative:	No P are Q
Particular Affirmative:	Some P are Q
Particular Negative:	Some P are not Q

16) Fallacies

$(P \rightarrow Q) \land Q$	$\therefore P$	Affirming the consequent or fallacy of the converse

$(P \rightarrow Q) \wedge \sim P \quad \therefore \sim Q$ Fallacy of the inverse